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Solution of the Reactor Point Neutron Kinetic Equations with Temperature Feedback Control Using MATLAB-Simulink Toolbox

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Abstract

Point kinetics equations are a system of stiff nonlinear ordinary equations. The solution of these equations provides a full understanding of neutron population's behavior, which helps controlling the criticality of a nuclear reactor. Solving point kinetics equations numerically has been always challenging due to the stiffness problem. In this paper, a solution for one-group delayed neutrons with negative temperature feedback using Simulink toolbox of MATLAB is presented. The solution is applied to the kinetics equation using initial step reactivity $\boldsymbol{\rho_0}$=0.2β. A GUI controller is built to control the model's input parameters. The results of the system are presented and compared with numerical methods. The variation along time of reactivity, temperature, and neutron density is analyzed.

Keywords: Point kinetic Equation, Simulink MATLAB, Negative Temperature feedback, One-group delayed neutrons

1. Introduction

The study of neutron population is of great importance in nuclear reactor kinetics. That is because it plays an essential role in sustaining the fission reactions in a nuclear reactor and it is a priority in nuclear safety .In the start-up process of a reactor, changes in the external source of neutrons have a significant influence on the reactor power. If control rods are withdrawn in a discontinued manner, the reactivity will increase and will cause a high increase in reactor power resulting in an over pressurized reactor core. To study the behavior of neutron population, one must solve the reactor point kinetics equations (PK). These equations are a system of stiff non-linear ordinary equations. They express the time-dependence of neutron population, and the change of reactivity and temperature within the reactor due to a change in neutron density and delayed neutron precursor concentrations. A change in neutron density affects the system leading to a "temperature-reactivity feedback".

In previous works, various methods were attempted for solving the system. Hend et alii (2013) [1] presented an analytical solution for the point kinetic equation with one group delayed neutron in the presence of Newtonian negative temperature feedback for different step reactivities. SCM (Stiffness Confinement Method) was used to solve the system and to analyze accidents in multiple types of reactors, e.g. thermal reactor such as HTR-M, and fast reactor like PRISM. Nahla and Zayed (2010) [2] have introduced an analytical approximation and numerical solution of the point nuclear reactor kinetic equations with average one-group of delayed neutron and temperature feedback. The analytical approximation is based on transforming the differential equations with respect to time into differential equations with respect to reactivity. The numerical solution is based on Taylor's Series method. Hadad et alii (2004) [3] have developed a MATLAB/Simulink model for six groups delayed neutrons with temperature reactivity feedback and input reactivity of 1.2$ and 3$. Rashid (2015) [4] reduced the point kinetic equation of six group delayed neutron equations into one effective delayed neutron group with and without temperature feedback and presented them as Simulink model. The model consists in two subsystems, prompt neutron subsystem and delayed neutron subsystem at constant neutron precursor concentration with an initial step reactivity of 0.3 β. Hend et alii (2014) [5] presented a solution for point kinetic equations with one group of delayed neutron using FORTRAN and MATLAB codes, that focused on the prompt super-critical process under two cases for three initial power inputs. First case is large step reactivity insertion of 1.5 β and 2 β. Second case is small step reactivity insertion of ρ_0= (β/4) and ρ_0= (β/1.2). Other reactivity shapes such as ramp and sinusoidal were tested as well.

In this work, we present a complete, precise, and clearer model of the point kinetics equations with one-group delayed neutrons and negative temperature feedback using Simulink toolbox of MATLAB. A GUI controller is built using MATLAB script to easily change the input parameters of the model. The results of the system are discussed and benchmarked with previous works [1] [2].

2. MATLAB-Simulink application point kinetic equation solution

The point kinetics equations are stiff nonlinear ordinary differential equations, which are written in the following form along with one-group delayed neutrons equation [2]:

$$\frac{dn}{dt} = \frac{\rho(t) - \beta}{\Lambda} n(t) + \lambda C(t) \quad (1)$$

$$\frac{dc(t)}{dt} = \frac{\beta}{\Lambda} n(t) - \lambda C(t) \quad (2)$$

Where n(t) is the neutron density (or power in MW), t is the time, ρ(t) is the reactivity, β is the fraction of delayed neutrons, λ is the decay constant, C(t) is the density of delayed neutron precursors (or latent power in MW), and Λ is the generation time.

Negative temperature feedback is also considered as part of the system [2]:

$$\rho(t) = \rho_0 - \alpha[T(t) - T_0] \quad (3)$$

$$\frac{dT(t)}{dt} = K_c n(t) \quad (4)$$

where T(t) is the temperature of the reactor (K), T_0 is the initial temperature of the reactor (K), K_c is the reciprocal of the thermal capacity of the reactor (K/MW*sec), and α is the temperature coefficient of reactivity (K^{-1}).

Initial condition of delayed neutron precursor density is introduced through the following form [2]:

$$C(0) = \frac{(\beta - \rho_0)}{\Lambda\lambda} n_0 \quad (5)$$

Equations 1 to 5 are shown in Figure (1)

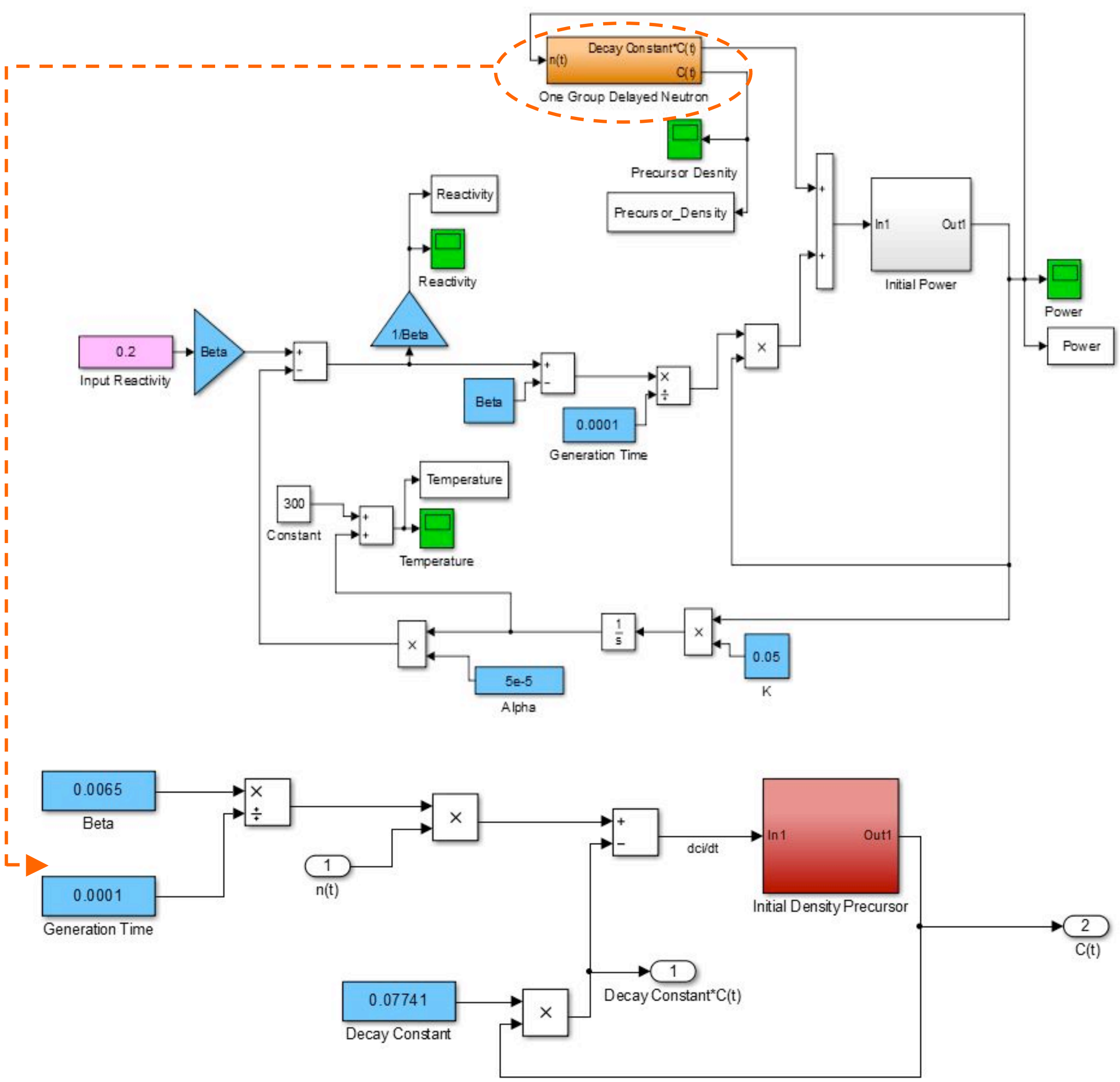

Fig. 1. Simulink modeling of point kinetic equation with negative temperature feedback and detailed subsystem of one group delayed neutrons

2.1 Simulink Model

Our Simulink model provides a complete and clearer representation of the point kinetics equations for one-group delayed neutron with negative temperature feedback. It clarifies the variables and constants used in solving the system. The main part of the system represents Equations 1, 3 and 4. Equations 3 and 4 are for “Negative temperature feedback.” The subsystem “One-Group Delayed Neutron” is a representation of Equations 2 and 5. The subsystem “1/s” is the initial condition of delayed neutron precursor density. Variables that can be controlled through GUI are the initial step reactivity, initial condition of precursor density, initial power and simulation time. Simulation results can be viewed directly through scope, and indirectly using MATLAB script through “to Workspace” block.

2.2 Graphical User Interface GUI

Graphical user interface was built as an additional tool that helps more in understanding and using the Simulink model. Figure (2) shows the main window of a small step reactivity GUI: it includes all the main parameters in the Simulink model. The interface interacts with the system so that the user can change the desired parameters and obtain different outcomes like reactivity, power and temperature plots. Figure (3) shows the detailed interface of the parameters window. The additional parameters of the Simulink model are in this window. Also, the initial power can be modified to get different graphical outcomes. Different scenarios in Figure (3) can also be added to the model and the user can move from one scenario to another in a very convenient way.

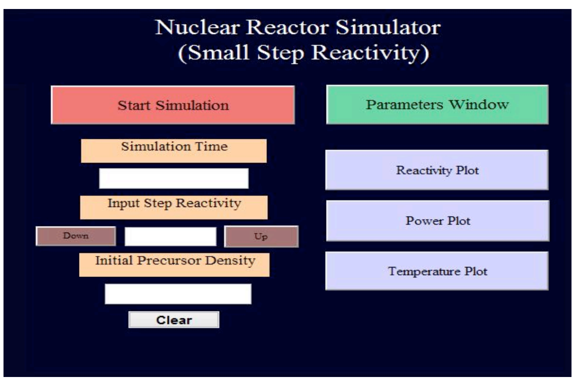

Fig. 2. GUI for step reactivity insertion

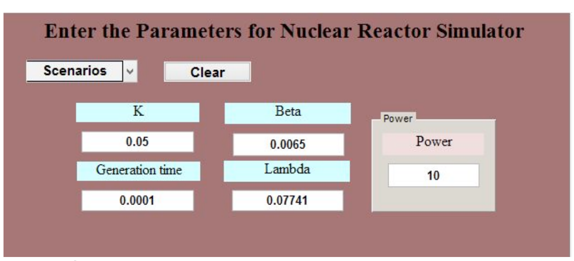

Fig. 3. GUI parameters window

3. Simulation discussion and benchmarking

The solution of point kinetic equation is preformed using Simulink toolbox of MATLAB. The results are compared against other methods: SCM (Stiffness Confinement Method) [1] and Taylor’s series method [2].

It is assumed that the input parameters are as following: λ=0.07741 sec^{-1}, β=0.0065, Λ=0.0001 sec, K=0.05 K/(MW sec), α =5e^{-5} K^{-1}, n(0)=10 MW, ρ_0=0.2β and C(0)=6717.5 MW, T(0)=300 K. In the start-up subcritical process, the external source of neutrons is important and the effect of temperature feedback is neglected because of reactor core’s low average temperature. During this process, neutron density is taken to full power and expected to increase to an initial maximum level due to the discontinued withdrawal of control rods. When step reactivity is suddenly inserted into the reactor at t=0, the prompt supercritical process starts taking place. The power of this process increases abruptly, and the maximum level of power is always larger than the initial power.

In Figure (4), it can be seen that reactivity begins decreasing slowly with time due to the effect that control rods have on neutron population. At t>0, changes in reactivity act back on neutron density with negative temperature feedback slowly influencing the system.

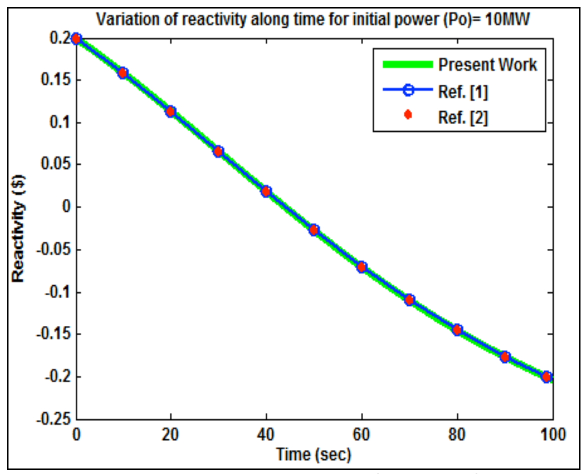

Fig. 4. Reactivity variation along time for ρ_0= 0.2β

In Figure (5), when step reactivity is inserted in the reactor core (withdrawing control rods), temperature rises slowly with time as a result of an increment in neutron population and fission chain reactions.

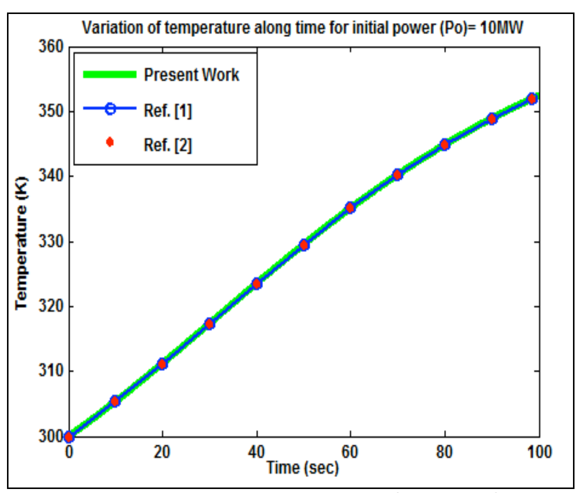

Fig. 5. Temperature variation along time for ρ_0= 0.2β

Table. 1. Variation of Reactivity, temperature and neutron density along time for initial step reactivity ρ_0= 0.2β.

Time (sec)	Reactivity ($)			Temperature (K)			Neutron Density (MW)		
	Present Work	Ref [1]	Ref [2]	Present Work	Ref [1]	Ref [2]	Present Work	Ref [1]	Ref [2]
0	0.2	0.2	0.2	300	300	300	10	10	10
10	0.159	0.1589	0.158976	305.3	305.3	305.333	11.271	11.28	11.270597
20	0.1139	0.1139	0.113885	311.19	311.2	311.195	12.097	12.11	12.094137
30	0.0666	0.06659	0.06661	317.3	317.3	317.341	12.4091	12.4	12.402006
40	0.0191	0.0191	0.019115	323.5	323.5	323.515	12.2248	12.22	12.220168
50	-0.0269	-0.02687	-0.026878	329.49	329.5	329.494	11.6418	11.63	11.639050
60	-0.0701	-0.07006	-0.070057	335.1	335.1	335.107	10.778	10.78	10.777035
70	-0.1096	-0.1096	-0.109569	340.2	340.2	340.244	9.7628	9.751	9.750800
80	-0.145	-0.145	-0.14498	344.8	344.8	344.847	8.6599	8.659	8.658279
90	-0.1762	-0.1762	-0.176181	348.9	348.9	348.904	7.5733	7.573	7.572253
98.7	-0.2	-0.2	-0.2	352	352	352	6.6757	6.673	6.671694

In Figure (6), the neutron density of the delayed supercritical process increases largely reaching a maximum level. As the negative temperature feedback becomes more and more pronounced, neutron density starts dropping with time.

The comparisons of the results obtained between time, reactivity, temperature, and neutron density using Simulink with SCM [1] and Taylor's series [2] are presented in Table 1. The results indicate excellent agreement with the previously published ones [1] [2].

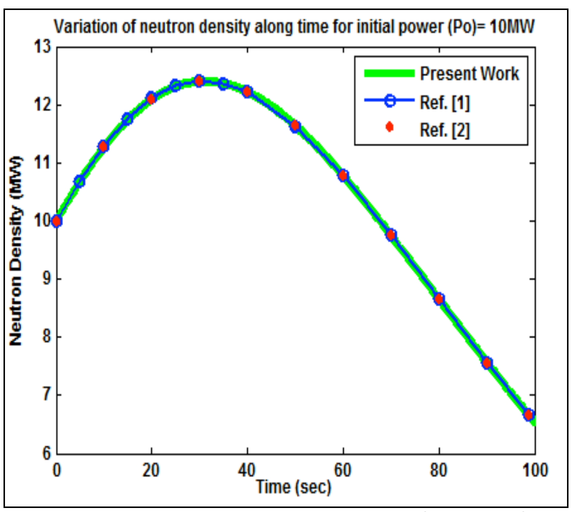

Fig. 6. Neutron density variation along time for ρ_0= 0.2β

5. Conclusions

Simulink model of point kinetic equation for one-group delayed neutrons with negative temperature feedback is designed to solve the point kinetic equation. The simulated results are plotted, tabulated, and compared with previous works [1] [2] that utilized numerical methods. The comparison of the Simulink model proposed shows excellent agreement with the results of previous works [1] [2] proving that the model well responds and interprets correctly the physics. This model can be integrated with different scenarios or fuzzy logic tools modeling the position of control rods. A six group delayed neutrons model has been also developed increasing the accuracy of the solutions: the model will be presented in details in the extended version of the paper.

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References

[1] Hend M. Saad, M. Aziz, H. M. Mansour (2013). "Analysis of Reactivity–Initiated Accident for Control Rods Ejection", Journal of Nuclear and Particle Physics, 45-54.

[2] A.A Nahla, E. M. E. Zayed, "Solution of the nonlinear point nuclear reactor kinetics equations", Prog. Nucl. Energy, P. (1-4), (2010).

[3] K.Hadad, A.Mohamadi, H.Sabet, N.Ayobian, M.Khani (2004). "Numerical Solution of Multigroups Point Kinetic Equations by Simulink Toolbox of Matlab Software". 2nd International Conference on Nuclear Science and Technology, (pp. 1-4). Iran.

[4] N. Rashid (2015). "Modeling Nuclear Processes by Simulink", in AIP Conference Proceedings, Selangor, Malaysia.

[5] Hend M. Saad, M. Aziz, H. M. Mansour (2014). "Analysis of Reactivity accident for control rods withdrawal at the thermal reactor". International Journal of Recent advances in Physics (IJRAP), 21-40.